

## Lanchester's Law: Too Few American Soldiers?

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By John Allen Paulos  
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Whatever one's opinion about the justification (or lack thereof) for the war, the failure of the Iraqi regime to collapse immediately has surprised many people.

To some it indicates that the American military strategy is based on excessively optimistic assessments. Retired general turned TV pundit Barry McCaffrey, for example, recently stated that Defense Secretary Donald Rumsfeld has sent too few troops and too little equipment to Iraq to dislodge Saddam Hussein.

Struck by such second thoughts among some in the military, I offer the following marginal contribution to the discussion.

It concerns Lanchester's Square Law, which was formulated during World War I and has been taught in the military ever since. Used to model conflicts from ancient times to the Battle of Trafalgar and Iwo Jima, it may be relevant to the Iraq situation as well.

Although usually couched in terms of differential equations (the context in which I first came across it), Lanchester's Law can be paraphrased as follows: "The strength of a military unit – planes, artillery, tanks, or just soldiers with rifles – is proportional not to the size of the unit, but to the square of its size."

What does this mean?

### The Effect of Quantity

Before returning to its application to Iraq, let me illustrate Lanchester's Law with a simplified conflict between two armies, denoted army QN (for quantity) and army QL (for quality), each of which has 500 pieces of artillery.

Assume furthermore that the two sides' artilleries are more or less equivalent in effectiveness and are capable of destroying each other at a rate

of, say, 6 percent per day. That is, after one day each side will have 94 percent of what they had the day before.

Neither side has an advantage, but let's alter the balance of power in a way suggested by a nice example from a new book, *What the Numbers Say*, by Derrick Niederman and David Boyum. What happens if we assume that army QN can increase its artillery to 1,500 pieces, three times as many as army QL has?

There are two consequences. One is that each of QL's artillery pieces will receive three times as much fire from QN's artillery as before because QN now has three times as many guns as QL. Because of this QL will lose artillery at a rate of 18 percent ( $3 \times 6$  percent) per day.

The other consequence is that each piece of QN's artillery will receive one third as much fire from QL's artillery as before because QL now has one third as many guns as QN. Because of this QN will lose artillery at a rate of 2 percent ( $1/3 \times 6$  percent) per day.

Lanchester's Law in this case: Tripling the number of pieces of army QN's artillery leads to a nine-fold advantage (18 percent versus 2 percent) in its relative effectiveness.

### The Effect of Quality

Armies can increase not only the number of their artillery (or planes, tanks, or soldiers) but can also increase their quality, and so we alter the balance of power again.

Let's assume that army QL counters QN's numerical superiority with better technology. It does so by upgrading its 500 pieces of artillery to make them each 9 times as accurate as QN's 1,500 pieces of artillery.

If QL can upgrade its artillery in this way, its rate of hitting QN's artillery (if QN has an equal number of pieces) will no longer be 6 percent per day, but 54 percent ( $9 \times 6$  percent). But because we're still assuming that QN has three times as many pieces of artillery, QL destroys only 18 percent ( $1/3 \times 54$  percent) of QN's artillery each day.

Moreover, the rate at which QN's artillery takes out QL artillery pieces remains the same as calculated above at 18 percent per day.

The bottom line: it takes a ninefold increase in QL's quality to make up for a threefold increase in QN's quantity. In general, it takes an  $N^2$ -fold increase in quality to make up for an  $N$ -fold increase in quantity.

### **In Iraq**

Now to the war where, in some limited respects, the Republican Guard and irregulars are QN and the American and British forces are QL. If the units under discussion have planes, cruise missiles and the like, there is no comparison and Lanchester's Law is not relevant. With tanks and artillery, Lanchester's Law does come into play, and American qualitative superiority again easily wins the day.

It's only when we get down to the level of individual soldiers with rifles in house-to-house fighting that the balance becomes unclear.

It's here that Lanchester's Law suggests that American soldiers' smaller degree of superiority may not always make up for a potential Iraqi numerical advantage (unless weapons more destructive than rifles are unleashed); it takes a big qualitative advantage to overcome a small quantitative one.

Of course, this analysis is necessarily very simplistic and ignores many other factors. Much depends, for example, on whether or not Iraqis are committed to the fight, something that's more likely if they're engaging an essentially all-American army rather than one broadly composed and under the aegis of the United Nations.

Whatever the duration of the war in Iraq, Lanchester's Law reminds us why urban guerilla warfare appeals to those with a weak conventional military. It is a worrisome fact that only at this level is there less scope for technological superiority.

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